# Qualifying Exam Problems (Electromagnetism)

**written by Prof. Intae Yu in March, 2016**

1. The atomic nucleus with charge $Ze$ is assumed to be a uniformly charged sphere of radius $a$.

(1) Write down the Poisson's equations for the scalar potential $\phi$ for $r \geq a$ and $r \leq a$ respectively.

(2) Calculate the potential $\phi$ by solving the Poisson's equation for $r \geq a$. The potential must vanish at $r = \infty$.

(3) Calculate the potential $\phi$ by solving the Poisson's equation for $r \leq a$. The potential must satisfy proper boundary conditions at $r = a$.

Hint: In spherical coordinates, the Laplacian operator $\nabla^2$ is written as $\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}) + \frac{1}{r^2\sin^2\phi}\frac{\partial^2}{\partial\theta^2} + \frac{1}{r^2\sin\phi}\frac{\partial}{\partial\phi}(\sin\phi\frac{\partial}{\partial\phi})$ .

2. Consider a sphere of radius $R$. There is a unit charge at the position r' outside the sphere. We want to calculate the Dirichlet Green function $G_D$(r, r') by requiring the electric potential to be zero on the spherical surface. The r vector denotes the point of observation.

(1) Using the image charge method, calculate the image charge inside the sphere and its position in terms of r' and $R$.

(2) Using the result of (1), write down the Green function $G_D$(r, r') as a function of $r$, $r'$ and $R$ and $\gamma$ where $\gamma$ is the angle between r and r'.

3. We consider a conducting sphere of radius $R$ consisting of two hemispheres. The upper hemisphere is kept at the potential V while the lower hemisphere is kept at the potential -V. We can use the Green function method to calculate the scalar potential $\phi$. The Dirichlet Green function $G_D$(r, r') is given as follows:

$$G_D(r,r') = \frac{1}{\sqrt{r^2 + r'^2 - 2rr'\cos\gamma}} - \frac{1}{\sqrt{r^2 r'^2/R^2 + R^2 - 2rr'\cos\gamma}}$$

The r vector denotes the position of observation and the r' vector denotes the position of charges where $\gamma$ is the angle between r and r'. Using the above Green function, calculate the potential along the positive z-axis.

4. In the presence of an external electric field, the polarization of molecules is induced and the electric field is modified accordingly.

(1) The surface charge density $\sigma_p$ and space charge density $\rho_p$ induced by the polarization P are given as $P \cdot \hat{n}$ and $-\nabla \cdot P$ respectively where $\hat{n}$ is a unit vector normal to the surface. Show that the total induced charges on the surface S and inside the volume V enclosed by S are zero. Discuss the physical implications.

(2) Molecular dipoles induce the polarization and modify an existing electric field. Do these dipoles contribute to the dielectric displacement D? Discuss the result and its physical implication.

5. An electrically neutral sphere, consisting of a dielectric medium with permittivity ε, is imbedded in a uniform electric field $E_0$. On the spherical surface, polarization charges are formed while the interior and the exterior of the sphere remain charge-free. Due to the surface polarization charges, the interior electric field $E_i$ and the exterior electric field $E_a$ are expected to behave differently. The permittivity outside the sphere is assumed to be 1.

(1) By applying proper boundary conditions, calculate the interior electric field $E_i$ and the polarization P and discuss the result.

(2) Calculate the exterior electric field $E_a$ and discuss the result.

Hint: The scalar potential $\phi$ by an dipole moment p is given by

$$\phi = \frac{p \cdot r}{r^3}.$$

6. A long straight, cylindrical conductor of radius $R$ carries a

current $I$ along its axis. This conductor has a cylindrical hole (radius $b$) whose axis is shifted by the distance $d$ from the axis of the conductor ($b+d<R$). Calculate the magnetic field intensity B inside the hole.

7. Consider a sphere of radius R consisting of a magnetic medium. The sphere is uniformly magnetized in the direction of z-axis ($\overrightarrow{\mathrm{M}}=\mathrm{M}e_z$). For r>R, $\nabla\times\mathrm{B}=0$ and $\nabla\cdot\mathrm{B}=0$. Therefore we can introduce the magnetic potential $\phi_{\mathrm{M}}$ such that $\mathrm{B}=-\nabla\phi_{\mathrm{M}}$ and we have the Laplace equation $\nabla^2\phi_{\mathrm{M}}=0$. Solving the equation, the general solution is given by $\phi_{\mathrm{M}}=\sum_{l=0}^{\infty}\alpha_l\frac{P_l(\cos\theta)}{r^{l+1}}$ where $P_l(\cos\theta)$ is the l-th Legendre polynomial.

(1) Calculate the magnetic moment m of the sphere.

(2) Assuming that B is uniform inside the sphere, calculate the B and H field inside and outside the sphere by applying proper boundary conditions on the spherical surface.

(3) Plot the H field lines and B field lines, respectively, and discuss the result.

8. A magnetizable sphere of permeability $\mu$ is in an external field $\mathrm{B}_0$. The interior fields $\mathrm{B}_i$ and $\mathrm{H}_i$ can be calculated as follows.

$$\mathrm{B}_i=\mathrm{B}_0+\frac{8\pi}{3}\mathrm{M},\quad \mathrm{H}_i=\mathrm{B}_0-\frac{4\pi}{3}\mathrm{M}$$

where M is the magnetization of the sphere.

(1) The material of the sphere is not ferromagnetic. Calculate the magnetization M, $\mathrm{B}_i$, and $\mathrm{H}_i$ in terms of $\mathrm{B}_0$ and $\mu$.

(2) If the material is ferromagnetic, discuss how we can determine M, $\mathrm{B}_i$, and $\mathrm{H}_i$.