1. 질량이 각각 m_1과 m_2인 두 입자가 1차원 탄성충돌하는 경우를 고려하자. 충돌 전의 속도가 각각 u_1과 u_2라고 하고, 충돌 후 속도가 v_1과 v_2라고 하자. 충돌 전후에 대하여 보존되는 물리량은 무엇인가. 그리고 보존되는 물리량으로부터 두 입자간 상대속도가 크기는 변하지 않고, 방향은 반대가 됨을 보이시오.

1. Consider the elastic collision of two particles of mass m_1 and m_2, respectively, in one dimension. Their respective velocities are u_1 and u_2 before the collision, and v_1과 v_2 afterwards. What are the conserved physical quantities in the collision process. Based on the conserved quantities, prove that the relative velocities of the particles remain constant in magnitude and reverses their signs in the collision process.

2. 질량이 m이고, 전하가 q인 구조 없는 점전하가 전자기장 $(\mathrm{E}(\mathrm{t},\mathbf{x}),\mathrm{B}(\mathrm{t},\mathbf{x}))$ 영향 아래 고전역학적인 2차원 운동을 하는 경우를 고려하자.

(a) 일정한 전기장이 z-축 방향으로만 걸려있는 경우, $\mathrm{E}(t,\mathbf{x})=\mathrm{E}_0\hat{\mathbf{z}}$, 가능한 운동의 형태는 무엇인가.

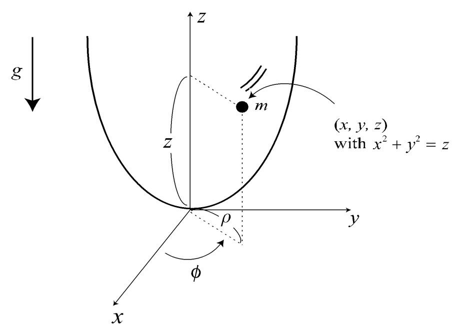
(b) 일정한 자기장이 z-축 방향으로만 걸려있는 경우, $\mathrm{B}(\mathrm{t},\mathbf{x})=\mathrm{B}_0\hat{\mathbf{z}}$, 가능한 운동의 일반적 형태를 뉴턴의 제 2법칙을 이용하여 구하시오.

2. A point particle of mass m and charge q is moving under the influence of electromagnetic fields $(\mathrm{E}(\mathrm{t},\mathbf{x}),\mathrm{B}(\mathrm{t},\mathbf{x}))$according to non-relativistic dynamics in two dimensions.

(a) When a constant electric field is present in the z-direction such that $\mathrm{E}(t,\mathbf{x})=\mathrm{E}_0\hat{\mathbf{z}}$, describe the possible motion of the charged particle.

(b) When a constant magnetic field is applied in the z-direction, $\mathrm{B}(\mathrm{t},\mathbf{x})=\mathrm{B}_0\hat{\mathbf{z}}$, describe possible motion of the particle by solving Newton's equation of motion.

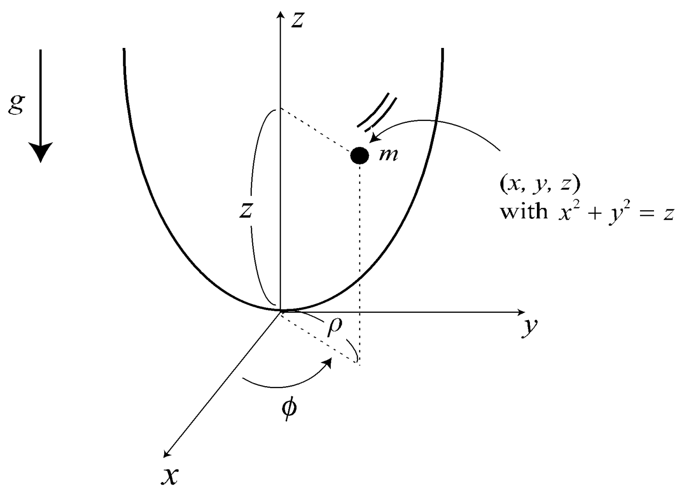
3. 질량이 m인 질점의 운동이 $x^2+y^2=z$로 표시되는 마찰이 없는 곡면을 따라 운동을 하는 경우를 생각하자. 가능한 외력은 중력으로 $\vec{g}=-g\hat{z}$인 중력 가속도를 갖는다.



(a) 이 입자의 운동을 기술하는 라그랑지안을 쓰시오.

(b) Euler-Lagrange 운동방정식을 구하시오.

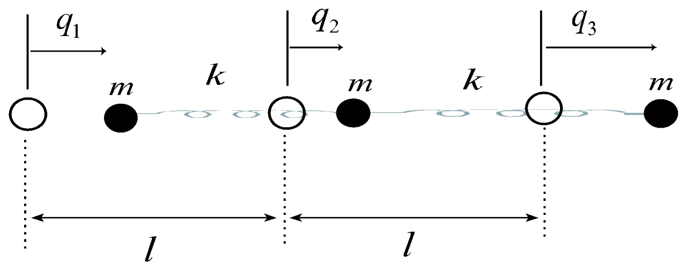
3. A point particle of mass m is moving on the frictionless surface governed by the relation $x^2+y^2=z$. An external force imposes a constant acceleration $\vec{g}=-g\hat{z}$ on the particle.



(a) Write down the Lagrange for the motion of this particle.

(b) Derive the Euler-Lagrange equation of motion.

4. 아래 그림과 같이 1차원 배열을 지닌 3원자 분자계의 고유 진동 방식(normal modes)을 고전역학을 이용하여 기술하려고 한다. 원자 1,2,3의 질량은 각각 $m_1=m_3=m$, $m_2=M$ 이며, 원자 1과 2, 2와 3사이는 용수철상수 k인 용수철에 의해 연결되어 있고 다른 상화작용은 없다. 평형을 이루고 있을 때 원자 1과 2, 2와 3사이의 거리는 각각 ℓ이고, 그 일차원적인 변위를 각각 q_1, q_2, q_3라고 하자. 다음 질문에 답하시오.



(a) 이 계의 진동운동을 기술하는 라그랑지안을 쓰시오.

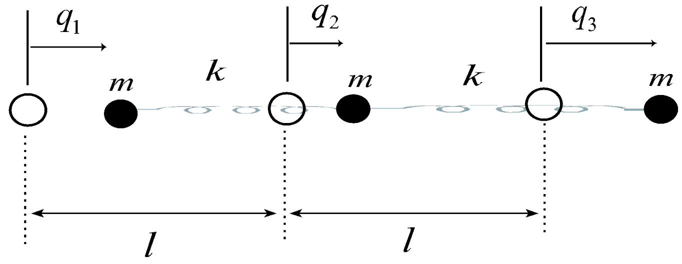
(b) 위에서 구한 라그랑지안으로부터 운동방정식을 구하시오.

(c) 이 원자들의 운동을 조화진동으로 보고, 그 고유진동수와 대응하는 고유변위를 구하시오.

(d) 위 문제에서 구한 고유진동수에 대응하는 고유 진동 방식을 그림으로 그리시오.

4. We want to describe the linearly coupled three-atom system such as shown below in classical mechanics and deduce their normal modes. Masses of the atoms are,

respectively, $m_1 = m_3 = m$, $m_2 = M$. Spring constants for atoms 1 and 2, and for 2 and 3, are both k. Distance between atoms 1 and 2, and atoms 2 and 3 are both given by ℓ. Displacements of each atom is denoted by q_1, q_2, q_3 . Answer the following questions.



(a) Write down the Lagrangian for the vibration motion of this coupled system.

(b) Derive the equations of motion based on the Lagrangian found in (a).

(c) Assuming a harmonic oscillator solution, find the characteristic frequency and the characteristic displacements.

(d) Draw schematic cartoons for each of the characteristic modes found in (c).

5. A particle of mass m moving with velocity $\vec{v}_1$ leaves a half-space in which its potential energy is a constant U_1 and enters another half-space in which its potential energy is a different constant U_2 . It then travels with constant velocity $\vec{v}_2$. Determine the change in the direction of motion of the particle. Express your answer in terms of $\sin\theta_1/\sin\theta_2$, where θ_1 and θ_2 are the angles between the normal of the plane and the velocities $\vec{v}_1$ and $\vec{v}_2$, respectively.

6. The Lagrangian concerning the motion of two particles which interact with the potential energy that depends only on the distance between them can be written as

$$L = \frac{1}{2} m_1 (\vec{v}_1)^2 + \frac{1}{2} m_1 (\vec{v}_1)^2 - U(|\vec{r}_1 - \vec{r}_2|).$$

(a) Simplify the Lagrangian by passing to the center-of-mass coordinates which satisfy $m_1 \vec{r}_1 + m_2 \vec{r}_2 = 0$. Express the Lagrangian using the relative coordinate $\vec{r} = \vec{r}_1 - \vec{r}_2$. What is the reduced mass m in terms of m_1 and m_2 ?

(b) Due to the conservation of angular momentum vector, the motion of $\vec{r}$ is confined to a plane. Express the reduced Lagrangian obtained in (a) in terms of the cylindrical coordinates (r, ϕ). How is the conserved angular momentum M_z expressed in terms of (r, ϕ)? The motion is assumed to take place in the xy plane.

(c) Using the result of (b), express the reduced Lagrangian obtained in (a) in terms of r and its time derivative dr/dt alone. What is the conserved energy in terms of r and dr/dt?

(d) Express the time t in terms of the integral of the function that depends on r. Express the angle ϕ in terms of the integral of another function that depends on r.

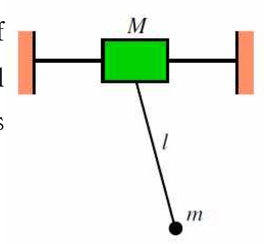
7. Which components of linear momentum $\vec{P}$ and angular momentum $\vec{L}$ are conserved in motion in the following fields? (a) the field of an infinite homogeneous plane, (b) that of an infinite homogeneous cylinder, (c) that of two points, (d) that of a homogeneous cone. Explain your answer.

8. A particle moves in a potential $V(r) = -V_0 e^{-\lambda^2 r^2}$.

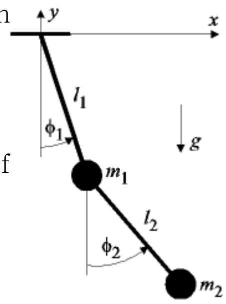
   (a) Given angular momentum l, find the radius of the circular orbit. (An implicit equation is fine here.)

   (b) What is the largest value of l for which a circular orbit exists? What is the value of $V_{eff}(r)$ at this critical orbit?

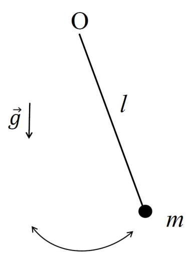
9. A pendulum of mass m and length l is hung from a support of mass M which is free to move horizontally on a frictionless rail (see the figure on right). (a) Find the equations of motion. (b) Is there a cyclic coordinate? If so, what is conserved?



10. Formulate the double-pendulum problem shown in the right Figure, in terms of Hamiltonian and Hamilton's equation of motion.
It is suggested you find the Hamiltonian by equation
$H(q,p,t) = \frac{1}{2}(\tilde{p} - \tilde{a}) T^{-1}(p-a) - L_0(q,t)$, where $\tilde{p}$ is the transpose of p, a column matrix.



11. As shown in the figure on the right, a point object with mass m is attached to a massless rigid string with length l and moves around a point O.
(a) Find the equations of motion for the object using the Lagrange's formalism. (b) Solve the equations motion and describe the motions of the object. (c) Obtain expression for the force of constraint.



12. For the tow-body central force problems, (a) Obtain the equivalent one-body problem

from the two-body central force problem? (b) Find the equations of motion and discuss the two first integrals for the equivalent one-body problem.

13. What is the rigid body? Show that only six coordinates are sufficient to specify the configuration of a rigid body.

14. What are the direct conditions for a (restricted) canonical transformation? How are these conditions related to the symplectic condition ($MJ\widetilde{M} = J$) for a canonical transformation? Here J is the antisymmetric matrix [$2n \times 2n$ square matrix composed of the $n \times n$ zero and unit matrices, $J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$] and M is a symplectic matrix.

15. What is the definition of Poisson brackets of two functions $u(q,p)$, $v(q,p)$ with respect to the canonical variables (q,p)? Show that all Poisson brackets are invariant under canonical transformations by using the symplectic condition ($MJ\widetilde{M} = J$), where J is the $2n \times 2n$ antisymmetric matrix, i.e. $J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$, where I is the $n \times n$ identity matrix.